图解张量(tensor)

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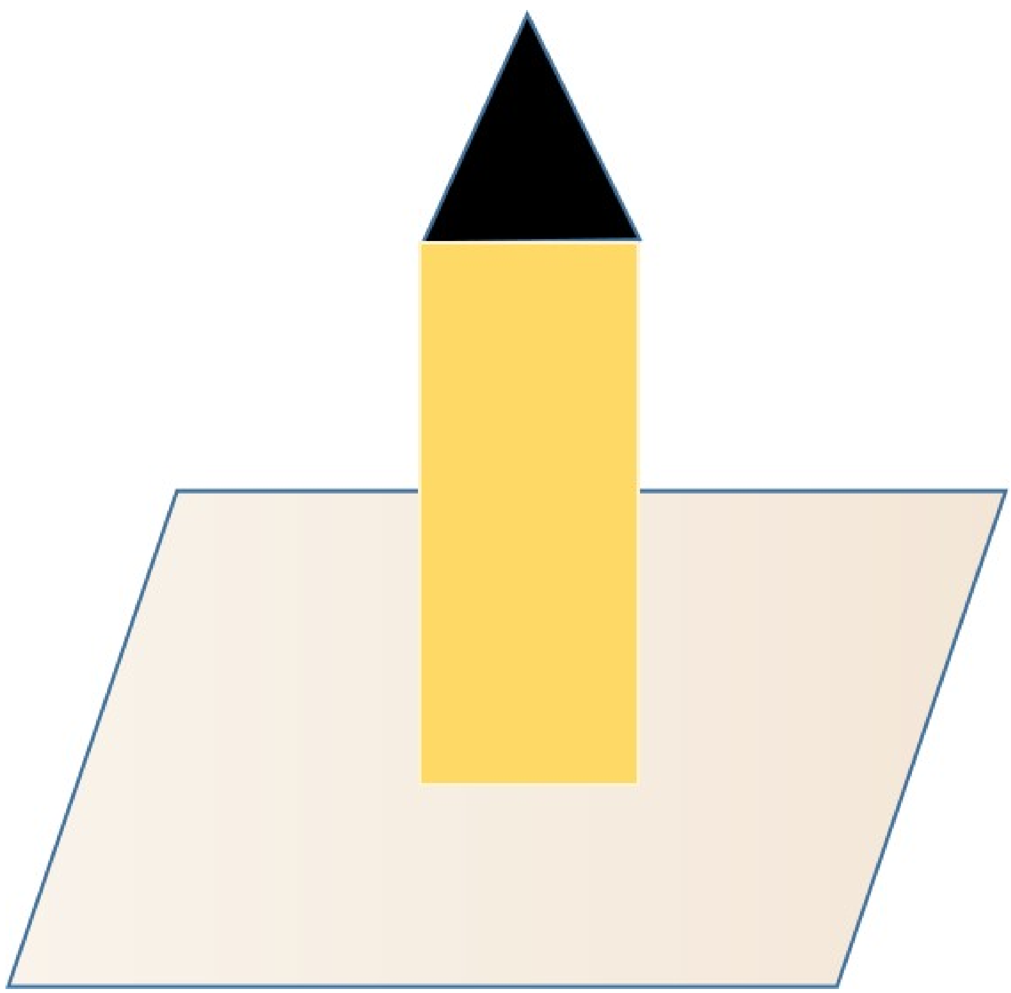
1 图解张量(tensor)

张量是什么？本讲的目的是回答这个问题，不是用一堆数学方程，用一些简单的家用物品，包括儿童积木、小箭头、几块纸板和一根尖头棒。我认为理解张量的最佳途径是首先确保你对向量的理解是扎实的。

What's a tensor? My goal for this video is to take about 12 minutes to answer that question, not using a bunch of mathematical equations, but instead some simple household objects including children's blocks, small arrows, a couple of pieces of cardboard, and a pointed stick. I think the very best route to understanding tensors is to begin by making sure that you're solid on your understanding of vectors.

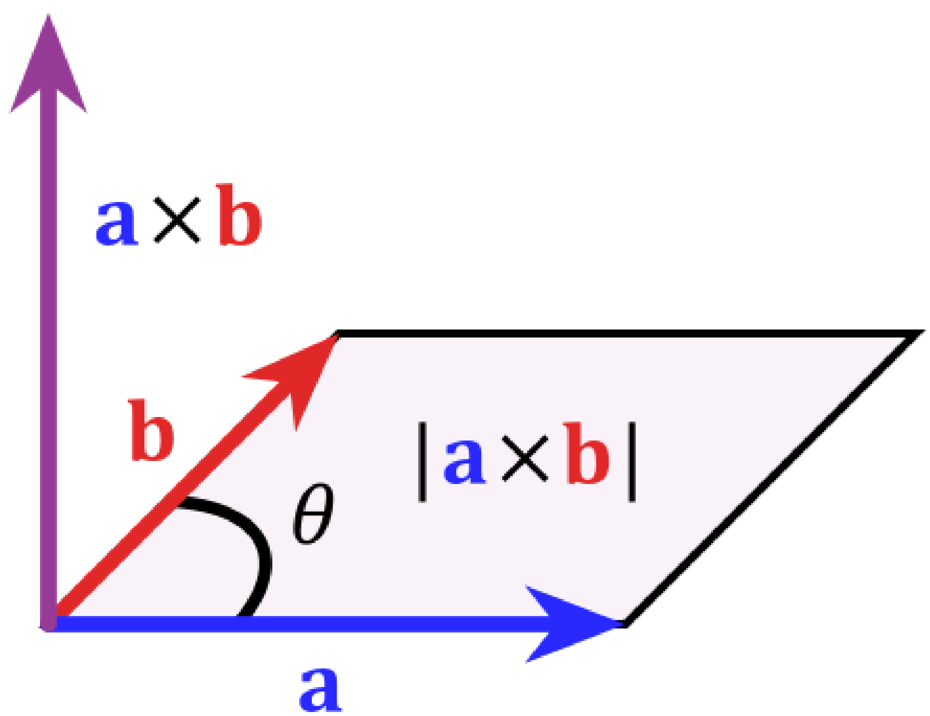


向量是表示同时具有大小和方向的量，其中箭头的长度与量的大小成比例，箭头的方向告诉您量的方向。这可以表示物体的重力，或地球磁场的强度和方向，或流动流体中粒子的速度。但向量也可以表示其他事物，例如面积。

If you've taken any college-level physics or engineering, representing a quantity that has both magnitude and direction, where the length of the arrow is proportional to the magnitude of the quantity and the orientation of the arrow tells you the direction of the quantity. This could represent the force of gravity on an object, or the strength and direction of the Earth's magnetic field, or the velocity of a particle in a flowing fluid. But vectors can represent other things as well, such as an area.

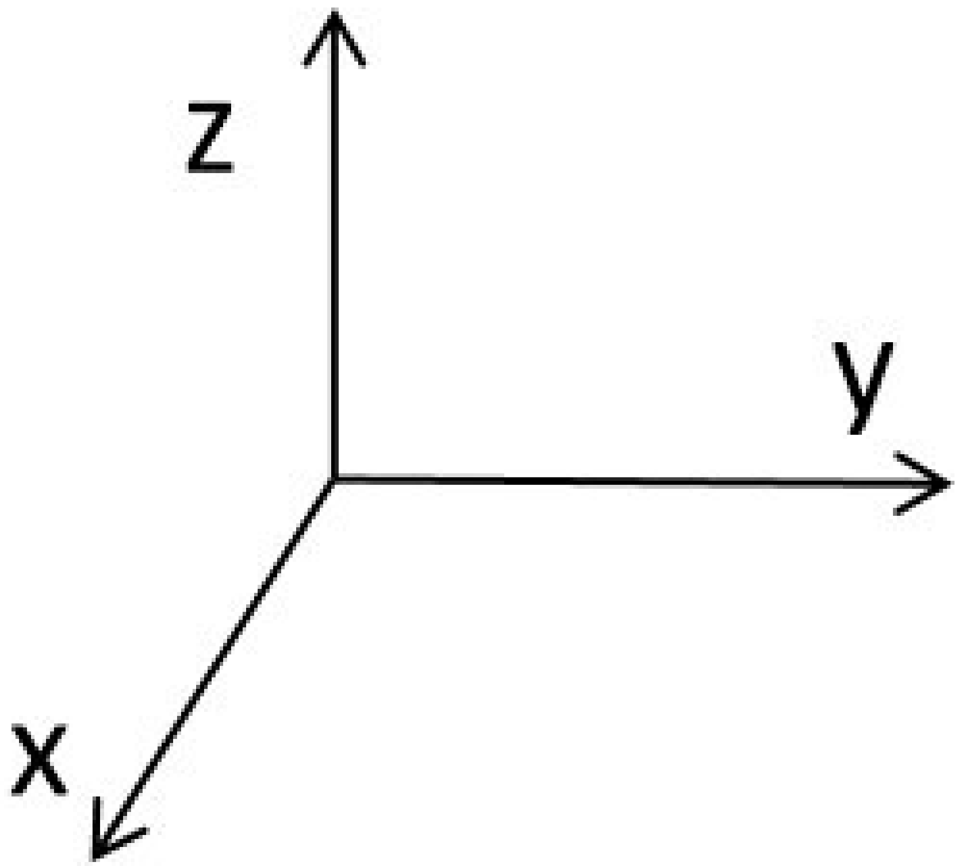
只需使向量的长度与面积（面积中的平方米数）成比例，然后使箭头的方向垂直于曲面。理解向量的叉乘就很容易理解用向量表示面积。

You simply make the length of the vector proportional to the amount of the area (the number of square meters in the area) and then you make the direction of the arrow perpendicular to the surface.



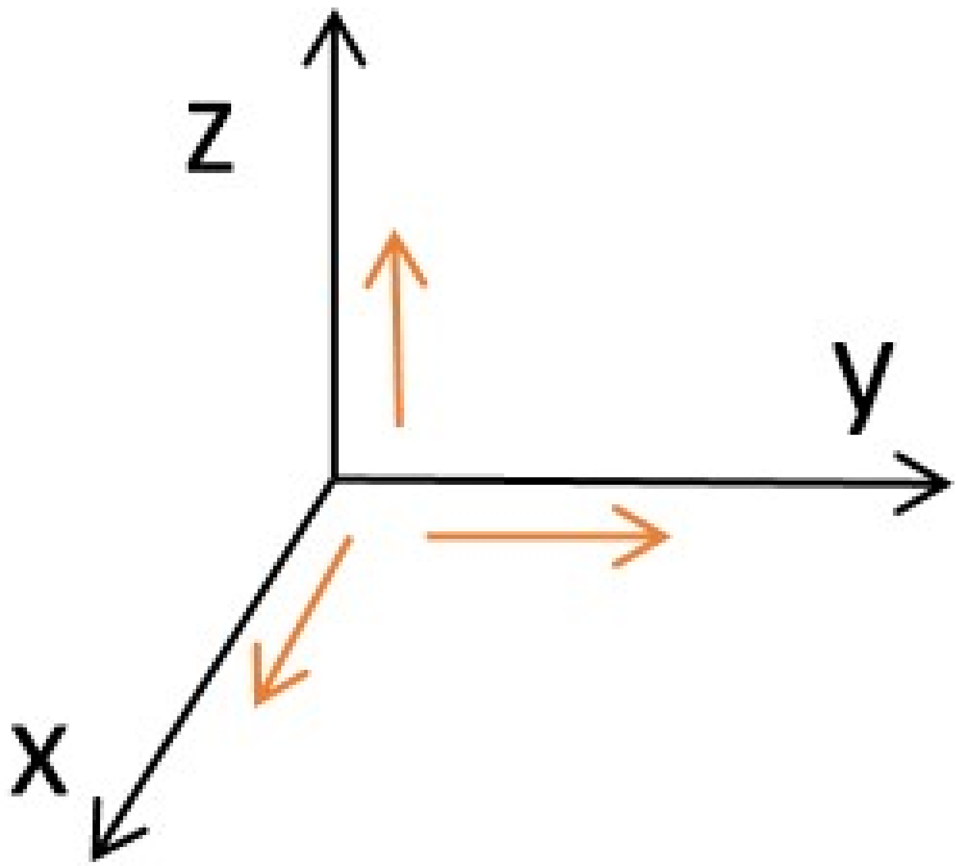
因此，这样也可以表示一个区域。所以向量可以代表很多东西。但是，如果除了把向量理解为表示数量的大小和方向的量，把向量理解为一类更广泛的称为张量的对象的成员，那么必须确保你理解向量分量和基向量，即向量的组成部分。

So in that way this can represent an area as well. So vectors can represent lots of things. But if you want to take the step beyond thinking of vectors representing quantities with magnitude and direction, to understanding that vectors are members of a wider class of object called tensors, then you have to make sure you understand vector components and basis vectors. If you're even going to think about the components of a vector, you better get yourself one of these.



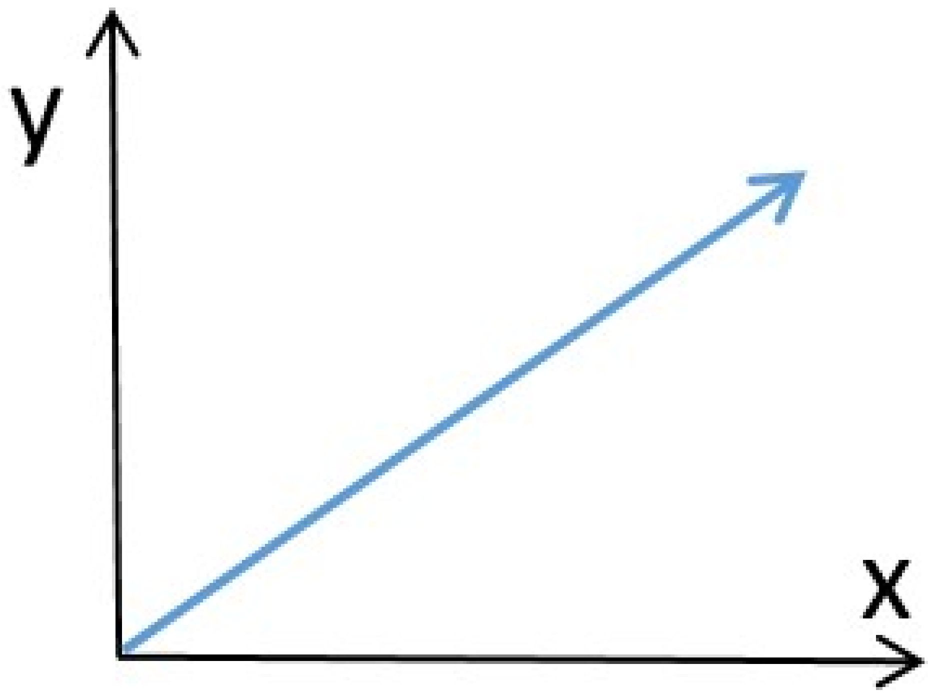
上图是笛卡尔坐标系(Cartesian coordinate system)。关于坐标系，需要记住的是，它们与坐标基向量一起出现，如下：

This represents a coordinate system - in this case I picked the simplest one with the x-axis the y-axis and z-axis all meeting at right angles. This represents the Cartesian coordinate system, and the thing to remember about coordinate systems is they come along with coordinate basis vectors.



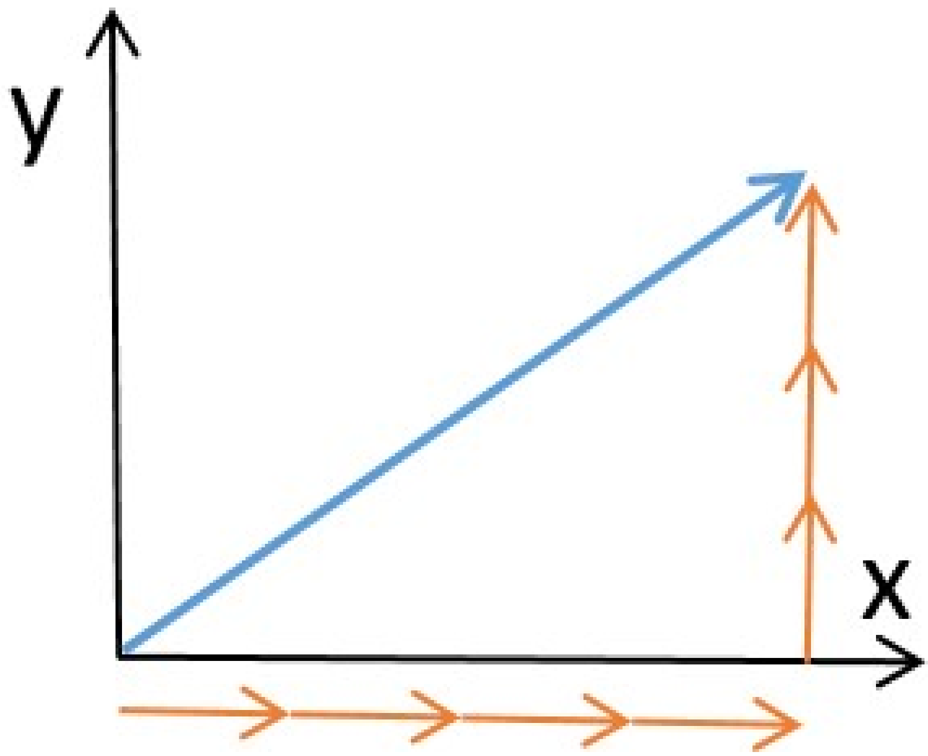
单位向量的长度是1，1表示要表达向量长度的任何单位之一。基向量或单位向量的方向是坐标轴的方向，因此这可能表示x方向上的单位向量$\hat{x}$或$\hat{\imath}$。一旦有了坐标系和单位向量，就可以找到向量的分量。

You probably ran into these as "unit vectors" and the thing to remember about these little guys is they have a length of one. One what? One of whatever the units are that you're going to express the length of your vector in. The direction of the basis vectors or unit vectors is in the direction of the coordinate axes, so this might represent the unit vector in the x direction that's often called "x" with a little hat over it or sometimes "i-hat". That's the x-hat unit vector - it points in the direction of increasing x coordinate. Likewise the y-hat (sometimes called the "j-hat") unit vector points in the direction of increasing y, and the z-hat or "k-hat" unit vector points in the direction of increasing z. Once you have the coordinate system and the unit vectors in place, now you're in a position to find the components of your vector.



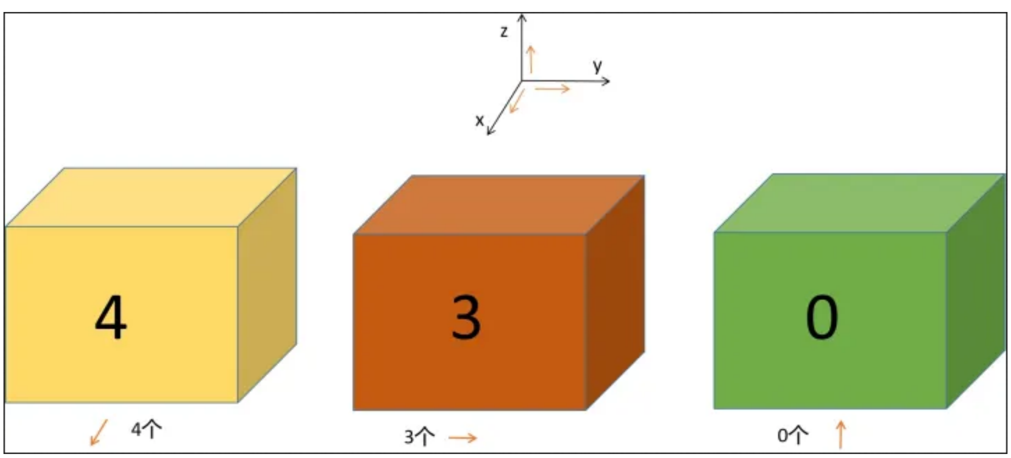
如果你从（x，y）平面上的向量开始。为了找到x分量，我将把这个向量投影到x轴上。为了找到y分量，我将把这个向量投影到y轴上。如果我使向量与x轴成更大的角度，请注意x分量越来越小，y分量越大。如果我让向量完全沿着x轴，那么阴影和向量的长度是相同的，在这种情况下，x分量就是向量的长度。

How exactly do you do that? I think it's easiest to understand how to find vector components if you begin with a vector in the (x,y) plane, so i'm going to lay this vector in the (x,y) plane at some angle to the x-axis. In order to find the x- component, I'm going to project this vector onto the x-axis. In order to find the y- component, I'm going to project this vector onto the y-axis. And how am I going to do those projections? project the vector onto the x- and y- axes. First I'm gonna shine the light perpendicular to the x-axis (that is parallel to the y-axis) and look for the shadow of the vector on the x- axis. That will be the x-component of this vector. As you can see the shadow of the vector on the x-axis ends right here. This is the x-component of this vector. If I make the vector have a greater angle to the x-axis, notice the shadow moves this way - the x-component is getting smaller. And if I make the vector lie entirely along the x-axis, then the shadow and the vector are the same length - the x-component is the length of the vector in that case. Now I've got my lights shining perpendicular to the y-axis (that is parallel to the x-axis) and the shadow cast by the vector onto the y-axis gives me the y-component of the vector. Notice that as I increase the angle to the x-axis and decrease the angle to the y- axis, the y-component is getting bigger.



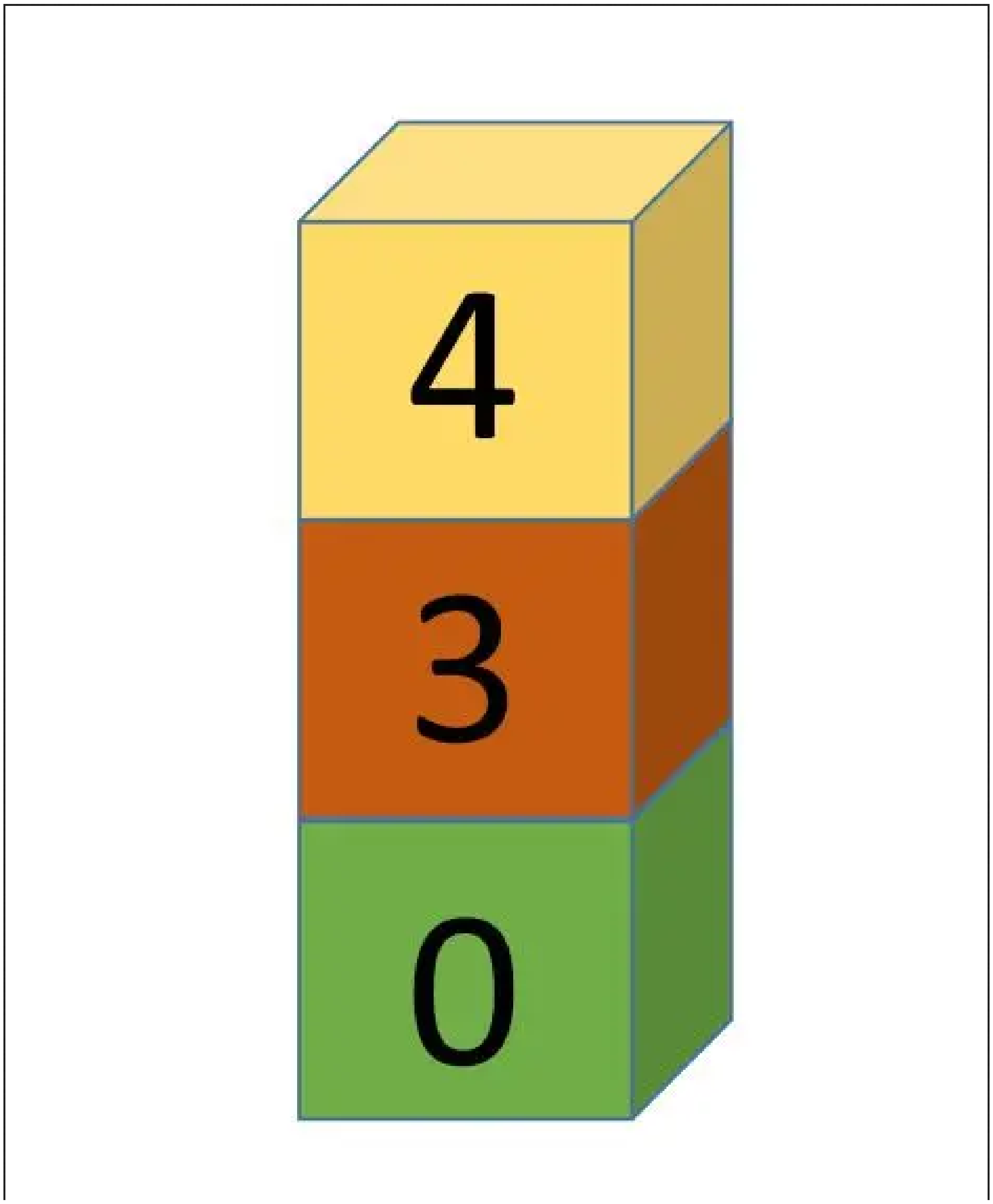
从向量的底端到向量的顶端，在x方向走多远，在y方向走多远？换言之，从这个向量的基部到顶端需要多少个x方向的单位向量（$\hat{x}$或$\hat{\imath}$）和多少个y方向的单位向量（$\hat{y}$或$\hat{\jmath}$）？这个向量由四个$\hat{x}$加上三个$\hat{y}$组成, 即$4\hat{x}+3\hat{y}$。

from the base of the vector to the tip of the vector, how far do I have to go in the x- direction and how far do I have to go in the y-direction?" In other words how many x-hat (or i-hat) unit vectors and how many y-hat (or j-hat) unit vectors would it take to get from the base to the tip of this vector? I can show you this if I get rid of these axes and just line up some x-hat basis vectors (these are going to go in the x-direction obviously), and some y-hat basis vectors. So in other words this vector is made up of about four x-hat plus three y-hat.



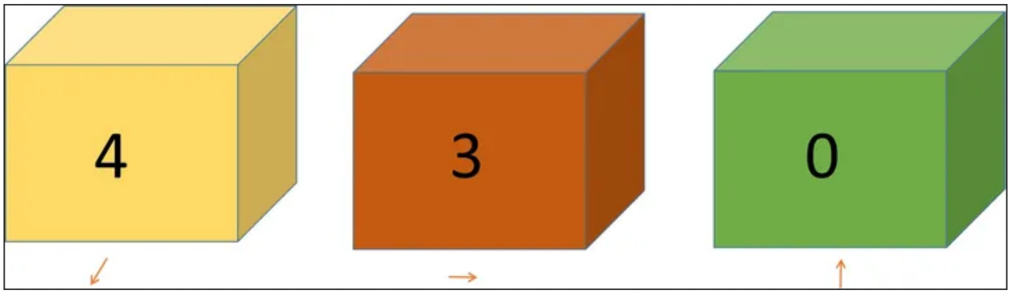
如果知道了基向量，这意味着不用为这个向量画箭头（基向量），你可以简单地说四个，再加上三个。如果加上z方向（因为这个向量没有z分量），z方向为0。换句话说，如果你知道基向量，用4，3，0这三个分量来表示该向量的一个完全有效的表示。

That means that instead of drawing an arrow for this vector you could simply say four of these, plus three of these. And if you want to be complete (since there's no z-component of this vector), zero of these. That is the same as this. In other words, this is a perfectly valid representation of that vector, and of course if you know the basis vectors, you wouldn't even have to put these on, would you?



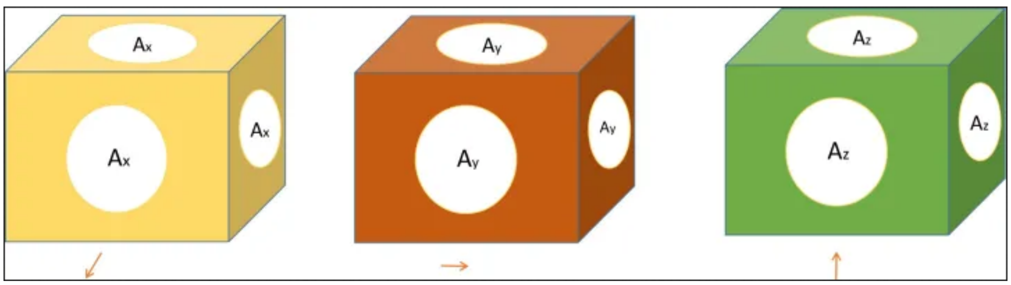
可以简单地使用这些分量作为向量。你可以把它写成一个小数组。如上图所示，甚至可以将它们堆叠起来，并在其周围加一组括号，这就像写列向量的方式。

You could simply use these components as your vector. You could write him in a little array. You could even stack them up, and put a nice set of parentheses around them. This looks just like the way you see column vectors written.



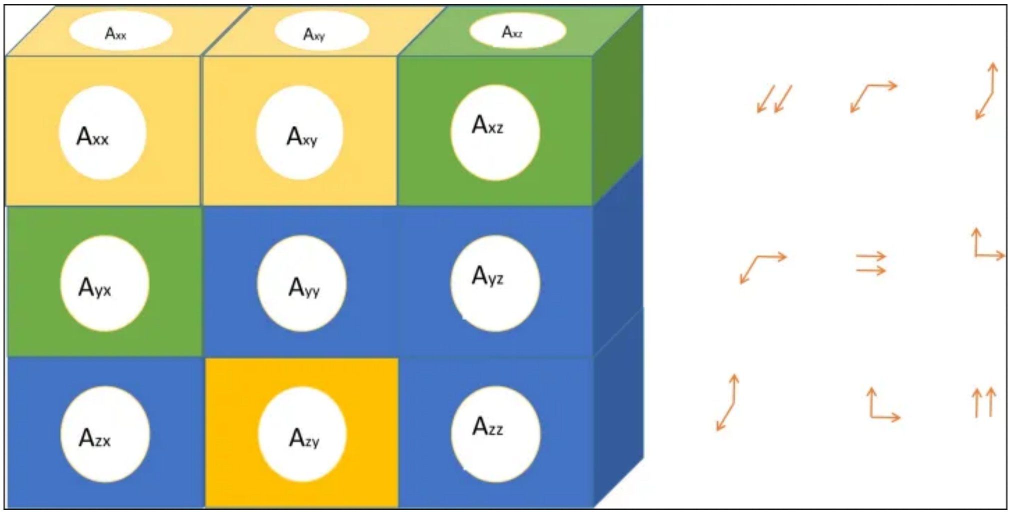
当然，这三个分量只与前面讲的那个向量有关。为了将其推广到一般的向量A，可以将这些分量替换为A_x、A_y和A_z。

Of course these three components pertain only to the vector we had lying on the table a minute ago. To generalize this to vector capital A, for example, we can replace these components with A sub x, and A sub y, and A sub z.



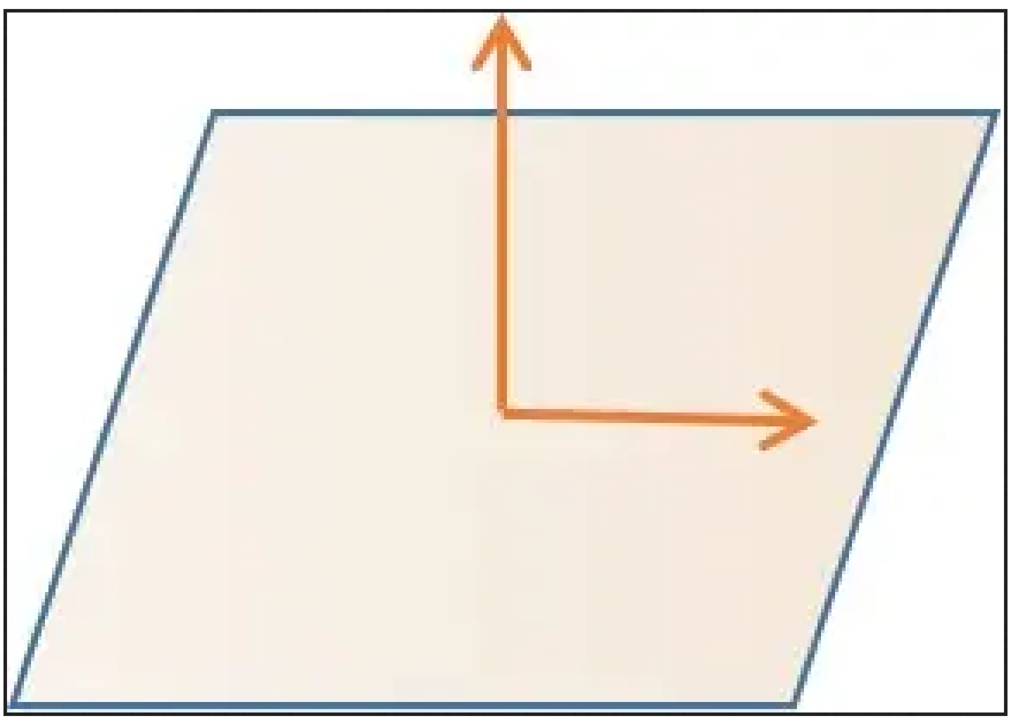
当然，Ax是属于基向量的分量，Ay是属于基向量的分量，Az是属于基向量的分量。注意，需要为每一个分量创建一个下标（index），因为每个分量只有一个方向指示器（即一个基向量）。这就是向量是“秩为一的张量(tensors of rank one)”的原因，一个下标，或每个分量一个基向量。同样，标量可以被看作零阶张量，因为标量没有方向指示器，因此不需要下标，这是零阶张量。马上就会明白为什么将张量表示为分量和基向量的组合如此强大，首先来看一些高阶张量的示例。

Of course, A sub x is the component that pertains to the x-hat basis vector, A sub y pertains to the y- hat basis vector, and A sub z pertains to the z-hat basis vector. Notice that we need one index for each of these, because there's only one directional indicator (that is one basis vector) per component. This is what makes vectors "tensors of rank one" - one index, or one basis vector per component. By the same token, scalars can be considered to be tensors of rank zero, because scalars have no directional indicators, therefore need no indices. Those are tensors of rank zero. I'll see in a minute why it's so powerful to represent tensors as this combination of components and basis vectors, but first I want to show you some examples of higher-rank tensors.



如上图所示，这是三维空间中二阶张量的表示。注意，现在没有三个分量和三个基向量，而是有九个分量和九组两个基向量。还请注意，分量不再具有单个下标，而是具有两个下标。为什么需要这样的表示呢？

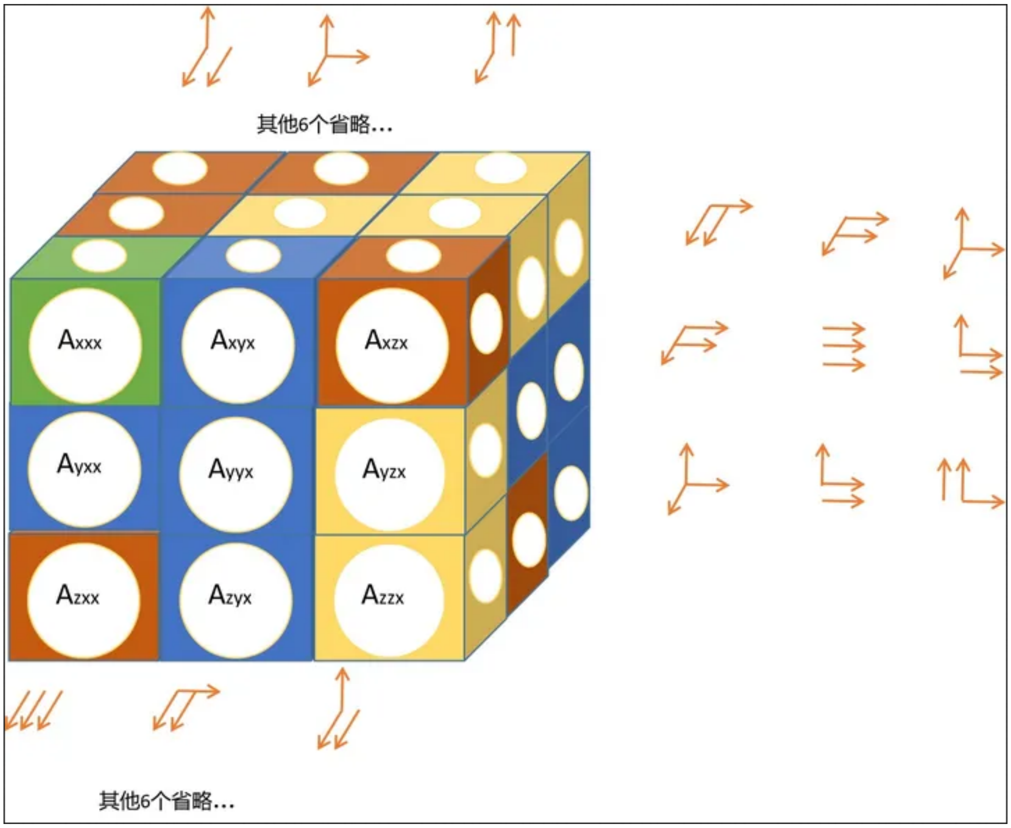
This is a representation of a rank-two tensor in three-dimensional space. Notice that instead of having three components and three basis vectors, we now have nine components and nine sets of two basis vectors. Notice also that the components no longer have a single index, they have two indices. Why might you need such a representation?



例如，考虑实体对象内部的力。在该对象内部，可以想象其面积向量指向x、y或z方向的曲面。在每一种类型的表面上，都可能有一个力，它的分量在x方向，或者在y方向，或者在z方向。因此，要充分描述所有可能曲面上的所有可能力，需要九个分量，每个分量都有两个下标表示基向量。例如，Axx可能指的是面积向量在x方向的曲面上的x方向力，Ayx可能指的是面积向量在y方向的曲面上的x方

向力，依此类推。九个分量和九组两个基向量的组合使其成为二阶张量。

Consider for example the forces inside a solid object. Inside that object you can imagine surfaces whose area vectors point in the x- or in the y- or in the z-direction. And on each of those types of surface, there might be a force that has a component in the x- or in the y- or in the z-direction. So to fully characterize all the possible forces on all the possible surfaces, you need nine components, each with two indices referring to basis vectors. So for example A sub xx might refer to the x-directed force on a surface whose area vector is in the x-direction, A sub yx might refer to the x-directed force on a surface whose area vector is in the y- direction, and so forth. This combination of nine components and nine sets of two basis vectors makes this a rank-two tensor.



上图是三维27个分量中三阶张量的表示，每个分量属于27组三个基向量中的一组。向量，Axyx属于两个x和一个y基向量，依此类推。整个前面板的第三个索引为x，属于这九组基本向量。中间的板都有y作为第三个索引，并且属于这九个，后面的面板都有z作为第三个索引，并且属于这九个。所以在三维空间中有27个分量，27组三个基向量，每个分量上有三个下标。为什么分量和基向量的组合让张量如此强大？答案是，在所有参考系中，所有观察者都同意这一点，不是在基础向量上，而是在分量和基向量的组合上。其原因是，**基向量在参考帧之间以一种方式变换，而分量也以这种方式变换，以便保持所有观察者的分量和基向量的组合相同。**正是张量的这一特性使得Lillian Lieber将张量称为“宇宙的事实(the facts of the universe)”。

This is a representation of a rank-three tensor in three-dimensional 27 components each pertaining to one of 27 sets of three basis vectors. vectors, A sub xyx pertains to two x and one y basis vector, and so forth. This entire front slab has x as the third index and pertains to these nine sets the basis vectors. The middle slab all has y as the third index and pertains to these nine, and the back slab all has z as the third index and pertains to those nine. So in three-dimensional space 27 components, 27 sets of three basis vectors, and three indices on each component. You may be wondering what is it about the combination of components and basis vectors that makes tensors so powerful. The answer is this all observers, in all reference frames, agree. Not on the basis vectors, not on the compliments, but on the combination of components and basis vectors. The reason for that is that the basis vectors transform one way between reference frames, and the components transform in just such a way so as to keep the combination of components and basis vectors the same for all observers. It was this characteristic of tensors that caused Lillian Lieber to call tensors "the facts of the universe". Thanks very much for your time.

A tensor is something that transforms like a tensor! 一个量, 在不同的参考系下按照某种特定的法则进行变换, 就是张量. 【本句出自：什么是张量 (tensor)？ - andrew shen的回答 - 知乎】